

Fractions



0650CH07

Chapter 7

7.1 Introduction

Subhash had learnt about fractions in Classes IV and V, so whenever possible he would try to use fractions. One occasion was when he forgot his lunch at home. His friend Farida invited him to share her lunch. She had five pooris in her lunch box. So, Subhash and Farida took two pooris each. Then Farida made two equal halves of the fifth poori and gave one-half to Subhash and took the other half herself. Thus, both Subhash and Farida had 2 full pooris and one-half poori.



2 pooris + half-poori—Subhash
2 pooris + half-poori—Farida

Where do you come across situations with fractions in your life?

Subhash knew that one-half is written as $\frac{1}{2}$. While eating he further divided his half poori into two equal parts and asked Farida what fraction of the whole poori was that piece? (Fig 7.1)

Without answering, Farida also divided her portion of the half puri into two equal parts and kept them beside Subhash's shares. She said that these four equal parts together make



Fig 7.1

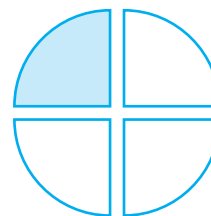


Fig 7.2

one whole (Fig 7.2). So, each equal part is one-fourth of one whole poori and 4 parts together will be $\frac{4}{4}$ or 1 whole poori.

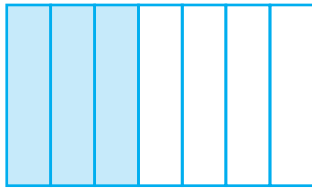


Fig 7.3

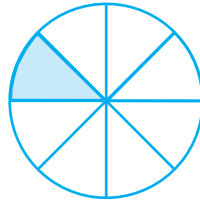


Fig 7.4

When they ate, they discussed what they had learnt earlier. Three parts out of 4 equal parts is $\frac{3}{4}$. Similarly, $\frac{3}{7}$ is obtained when we divide a whole into seven equal parts

and take three parts (Fig 7.3). For $\frac{1}{8}$, we divide a whole into eight equal parts and take one part out of it (Fig 7.4).

Farida said that we have learnt that **a fraction is a number representing part of a whole. The whole may be a single object or a group of objects.** Subhash observed that **the parts have to be equal.**

7.2 A Fraction

Let us recapitulate the discussion.
A fraction means a part of a group or of a region.

$\frac{5}{12}$ is a fraction. We read it as “five-twelfths”.

What does “12” stand for? It is the number of equal parts into which the whole has been divided.

What does “5” stand for? It is the number of equal parts which have been taken out.

Here 5 is called the numerator and 12 is called the denominator.

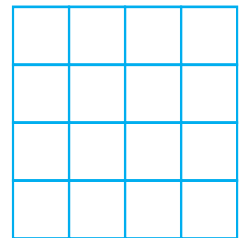
Name the numerator of $\frac{3}{7}$ and the denominator of $\frac{4}{15}$.

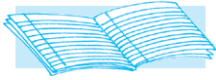
Play this Game

You can play this game with your friends.
Take many copies of the grid as shown here.

Consider any fraction, say $\frac{1}{2}$.

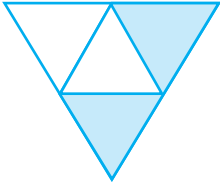
Each one of you should shade $\frac{1}{2}$ of the grid.





EXERCISE 7.1

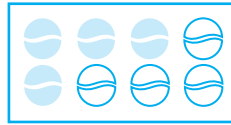
1. Write the fraction representing the shaded portion.



(i)



(ii)



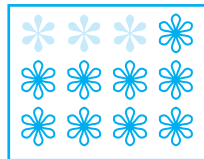
(iii)



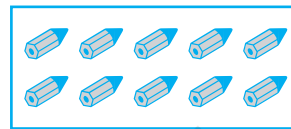
(iv)



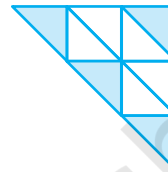
(v)



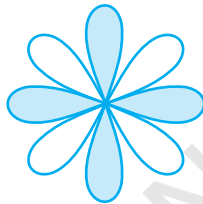
(vi)



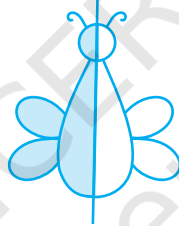
(vii)



(viii)



(ix)



(x)

2. Colour the part according to the given fraction.



(i) $\frac{1}{6}$



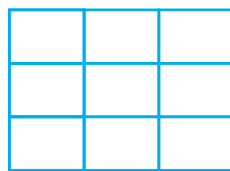
(ii) $\frac{1}{4}$



(iii) $\frac{1}{3}$

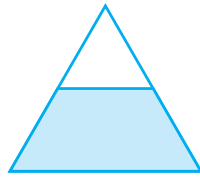


(iv) $\frac{3}{4}$

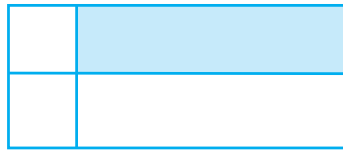


(v) $\frac{4}{9}$

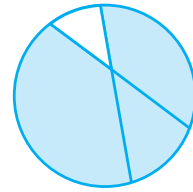
3. Identify the error, if any.



This is $\frac{1}{2}$

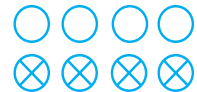


This is $\frac{1}{4}$



This is $\frac{3}{4}$

4. What fraction of a day is 8 hours?
 5. What fraction of an hour is 40 minutes?
 6. Arya, Abhimanyu, and Vivek shared lunch. Arya has brought two sandwiches, one made of vegetable and one of jam. The other two boys forgot to bring their lunch. Arya agreed to share his sandwiches so that each person will have an equal share of each sandwich.
 (a) How can Arya divide his sandwiches so that each person has an equal share?
 (b) What part of a sandwich will each boy receive?
 7. Kanchan dyes dresses. She had to dye 30 dresses. She has so far finished 20 dresses. What fraction of dresses has she finished?
 8. Write the natural numbers from 2 to 12. What fraction of them are prime numbers?
 9. Write the natural numbers from 102 to 113. What fraction of them are prime numbers?
 10. What fraction of these circles have X's in them?
 11. Kristin received a CD player for her birthday. She bought 3 CDs and received 5 others as gifts. What fraction of her total CDs did she buy and what fraction did she receive as gifts?



7.3 Fraction on the Number Line

You have learnt to show whole numbers like 0, 1, 2... on a number line.

We can also show fractions on a number line. Let us draw a number line and try to mark $\frac{1}{2}$ on it.

We know that $\frac{1}{2}$ is greater than 0 and less than 1, so it should lie between 0 and 1.

Since we have to show $\frac{1}{2}$, we divide the gap between 0 and 1 into two equal parts and show 1 part as $\frac{1}{2}$ (as shown in the Fig 7.5).

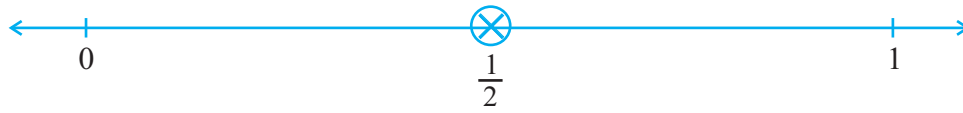


Fig 7.5

Suppose we want to show $\frac{1}{3}$ on a number line. Into how many equal parts should the length between 0 and 1 be divided? We divide the length between 0 and 1 into 3 equal parts and show one part as $\frac{1}{3}$ (as shown in the Fig 7.6)



Fig 7.6

Can we show $\frac{2}{3}$ on this number line? $\frac{2}{3}$ means 2 parts out of 3 parts as shown (Fig 7.7).



Fig 7.7

Similarly, how would you show $\frac{0}{3}$ and $\frac{3}{3}$ on this number line?

$\frac{0}{3}$ is the point zero whereas since $\frac{3}{3}$ is 1 whole, it can be shown by the point 1 (as shown in Fig 7.7)

So if we have to show $\frac{3}{7}$ on a number line, then, into how many equal parts should the length between 0 and 1 be divided? If P shows $\frac{3}{7}$ then how many equal divisions lie between 0 and P? Where do $\frac{0}{7}$ and $\frac{7}{7}$ lie?

Try These

1. Show $\frac{3}{5}$ on a number line.
2. Show $\frac{1}{10}$, $\frac{0}{10}$, $\frac{5}{10}$ and $\frac{10}{10}$ on a number line.
3. Can you show any other fraction between 0 and 1?
Write five more fractions that you can show and depict them on the number line.
4. How many fractions lie between 0 and 1? Think, discuss and write your answer?

7.4 Proper Fractions

You have now learnt how to locate fractions on a number line. Locate the fractions $\frac{3}{4}$, $\frac{1}{2}$, $\frac{9}{10}$, $\frac{0}{3}$, $\frac{5}{8}$ on separate number lines.

Does any one of the fractions lie beyond 1?

All these fractions lie to the left of 1 as they are less than 1.

In fact, all the fractions we have learnt so far are less than 1. These are **proper fractions**. A proper fraction as Farida said (Sec. 7.1), is a number representing part of a whole. In a proper fraction the denominator shows the number of parts into which the whole is divided and the numerator shows the number of parts which have been considered. Therefore, in a proper fraction the numerator is always less than the denominator.

Try These

1. Give a proper fraction :
 - (a) whose numerator is 5 and denominator is 7.
 - (b) whose denominator is 9 and numerator is 5.
 - (c) whose numerator and denominator add up to 10. How many fractions of this kind can you make?
 - (d) whose denominator is 4 more than the numerator.
(Give any five. How many more can you make?)
2. A fraction is given.
How will you decide, by just looking at it, whether, the fraction is
 - (a) less than 1?
 - (b) equal to 1?
3. Fill up using one of these : '>', '<' or '='

(a) $\frac{1}{2} \square 1$ (b) $\frac{3}{5} \square 1$ (c) $1 \square \frac{7}{8}$ (d) $\frac{4}{4} \square 1$ (e) $\frac{2005}{2005} \square 1$

7.5 Improper and Mixed Fractions

Anagha, Ravi, Reshma and John shared their tiffin. Along with their food, they had also, brought 5 apples. After eating the other food, the four friends wanted to eat apples.

How can they share five apples among four of them?



Anagha said, ‘Let each of us have one full apple and a quarter of the fifth apple.’



Anagha



Ravi



Reshma



John

Reshma said, ‘That is fine, but we can also divide each of the five apples into 4 equal parts and take one-quarter from each apple.’



Anagha



Ravi



Reshma



John

Ravi said, ‘In both the ways of sharing each of us would get the same share, i.e., 5 quarters. Since 4 quarters make one whole, we can also say that each of us would get 1 whole and one quarter. The value of each share would be five divided by four. Is it written as $5 \div 4$?’ John said, ‘Yes the same as $\frac{5}{4}$ ’. Reshma added that in $\frac{5}{4}$, the numerator is bigger than the denominator. The fractions, where the numerator is bigger than the denominator are called **improper fractions**.

Thus, fractions like $\frac{3}{2}$, $\frac{12}{7}$, $\frac{18}{5}$ are all improper fractions.

1. Write five improper fractions with denominator 7.
2. Write five improper fractions with numerator 11.

Ravi reminded John, ‘What is the other way of writing the share? Does it follow from Anagha’s way of dividing 5 apples?’

John nodded, ‘Yes, It indeed follows from Anagha’s way. In her way, each share is one whole and one quarter. It is $1 + \frac{1}{4}$ and written in short

as $1\frac{1}{4}$. Remember, $1\frac{1}{4}$ is the same as $\frac{5}{4}$.



This is 1
(one)



Each of these is $\frac{1}{4}$
(one-fourth)

Fig 7.8

Recall the pooris eaten by Farida. She got $2\frac{1}{2}$ pooris (Fig 7.9), i.e.

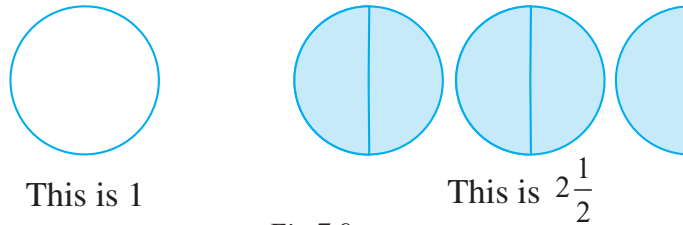


Fig 7.9

How many shaded halves are there in $2\frac{1}{2}$? There are 5 shaded halves.

So, the fraction can also be written as $\frac{5}{2}$. $2\frac{1}{2}$ is the same as $\frac{5}{2}$.

Fractions such as $1\frac{1}{4}$ and $2\frac{1}{2}$ are called **Mixed Fractions**. A mixed fraction has a combination of a whole and a part.

Do you know?
The grip-sizes of tennis racquets are often in mixed numbers. For example one size is ' $3\frac{7}{8}$ inches' and ' $4\frac{3}{8}$ inches' is another.

Where do you come across mixed fractions? Give some examples.

Example 1 : Express the following as mixed fractions :

- (a) $\frac{17}{4}$ (b) $\frac{11}{3}$ (c) $\frac{27}{5}$ (d) $\frac{7}{3}$

Solution : (a) $\frac{17}{4}$ $4 \overline{)17}$ i.e. 4 whole and $\frac{1}{4}$ more, or $4\frac{1}{4}$

$$\begin{array}{r} 4 \overline{)17} \\ - 16 \\ \hline 1 \end{array}$$

(b) $\frac{11}{3}$ $3 \overline{)11}$ i.e. 3 whole and $\frac{2}{3}$ more, or $3\frac{2}{3}$

$$\begin{array}{r} 3 \overline{)11} \\ - 9 \\ \hline 2 \end{array}$$

[Alternatively, $\frac{11}{3} = \frac{9+2}{3} = \frac{9}{3} + \frac{2}{3} = 3 + \frac{2}{3} = 3\frac{2}{3}$]

Try (c) and (d) using both the methods for yourself.

Thus, we can express an improper fraction as a mixed fraction by dividing the numerator by denominator to obtain the quotient and the remainder. Then

the mixed fraction will be written as $\text{Quotient} \frac{\text{Remainder}}{\text{Divisor}}$.

Example 2 : Express the following mixed fractions as improper fractions:

$$(a) 2\frac{3}{4} \quad (b) 7\frac{1}{9} \quad (c) 5\frac{3}{7}$$

Solution : (a) $2\frac{3}{4} = 2 + \frac{3}{4} = \frac{2 \times 4}{4} + \frac{3}{4} = \frac{11}{4}$

$$(b) 7\frac{1}{9} = \frac{(7 \times 9) + 1}{9} = \frac{64}{9}$$

$$(c) 5\frac{3}{7} = \frac{(5 \times 7) + 3}{7} = \frac{38}{7}$$

Thus, we can express a mixed fraction as an improper fraction as

$$\frac{(\text{Whole} \times \text{Denominator}) + \text{Numerator}}{\text{Denominator}}$$



EXERCISE 7.2

1. Draw number lines and locate the points on them :

$$(a) \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{4}{4} \quad (b) \frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{7}{8} \quad (c) \frac{2}{5}, \frac{3}{5}, \frac{8}{5}, \frac{4}{5}$$

2. Express the following as mixed fractions :

$$(a) \frac{20}{3} \quad (b) \frac{11}{5} \quad (c) \frac{17}{7}$$

$$(d) \frac{28}{5} \quad (e) \frac{19}{6} \quad (f) \frac{35}{9}$$

3. Express the following as improper fractions :

$$(a) 7\frac{3}{4} \quad (b) 5\frac{6}{7} \quad (c) 2\frac{5}{6} \quad (d) 10\frac{3}{5} \quad (e) 9\frac{3}{7} \quad (f) 8\frac{4}{9}$$

7.6 Equivalent Fractions

Look at all these representations of fraction (Fig 7.10).

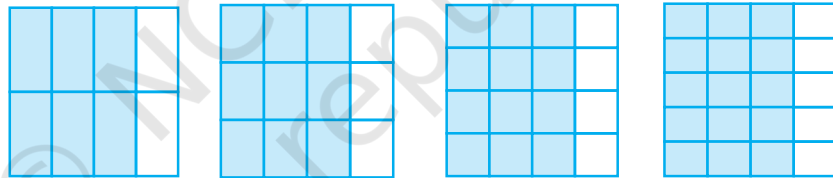


Fig 7.10

These fractions are $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$, representing the parts taken from the total number of parts. If we place the pictorial representation of one over the other they are found to be equal. Do you agree?

Try These

1. Are $\frac{1}{3}$ and $\frac{2}{7}$; $\frac{2}{5}$ and $\frac{2}{7}$; $\frac{2}{9}$ and $\frac{6}{27}$ equivalent? Give reason.
2. Give example of four equivalent fractions.
3. Identify the fractions in each. Are these fractions equivalent?



These fractions are called **equivalent fractions**. Think of three more fractions that are equivalent to the above fractions.

Understanding equivalent fractions

$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \dots, \frac{36}{72}, \dots$, are all equivalent fractions. They represent the same part of a whole.

Think, discuss and write

Why do the equivalent fractions represent the same part of a whole? How can we obtain one from the other?

We note $\frac{1}{2} = \frac{2}{4} = \frac{1 \times 2}{2 \times 2}$. Similarly, $\frac{1}{2} = \frac{3}{6} = \frac{1 \times 3}{2 \times 3} = \frac{1}{2}$ and $\frac{1}{2} = \frac{4}{8} = \frac{1 \times 4}{2 \times 4}$

To find an equivalent fraction of a given fraction, you may multiply both the numerator and the denominator of the given fraction by the same number.

Rajni says that equivalent fractions of $\frac{1}{3}$ are :

$$\frac{1 \times 2}{3 \times 2} = \frac{2}{6}, \quad \frac{1 \times 3}{3 \times 3} = \frac{3}{9}, \quad \frac{1 \times 4}{3 \times 4} = \frac{4}{12} \text{ and many more.}$$

Do you agree with her? Explain.

Try These

1. Find five equivalent fractions of each of the following:

- (i) $\frac{2}{3}$ (ii) $\frac{1}{5}$ (iii) $\frac{3}{5}$ (iv) $\frac{5}{9}$

Another way

Is there any other way to obtain equivalent fractions? Look at Fig 7.11.

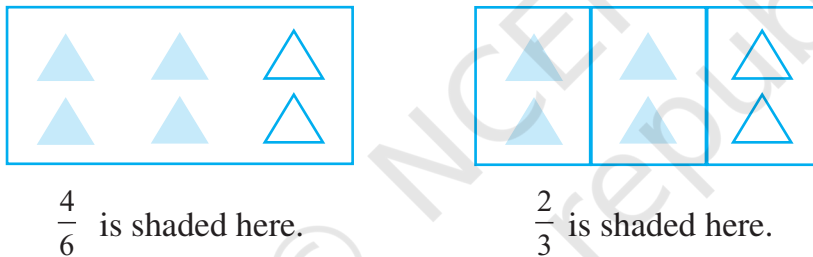


Fig 7.11

These include equal number of shaded things i.e. $\frac{4}{6} = \frac{2}{3} = \frac{4 \div 2}{6 \div 2}$

To find an equivalent fraction, we may divide both the numerator and the denominator by the same number.

One equivalent fraction of $\frac{12}{15}$ is $\frac{12 \div 3}{15 \div 3} = \frac{4}{5}$

Can you find an equivalent fraction of $\frac{9}{15}$ having denominator 5 ?

Example 3 : Find the equivalent fraction of $\frac{2}{5}$ with numerator 6.

Solution : We know $2 \times 3 = 6$. This means we need to multiply both the numerator and the denominator by 3 to get the equivalent fraction.

Hence, $\frac{2}{5} = \frac{2 \times 3}{5 \times 3} = \frac{6}{15}$; $\frac{6}{15}$ is the required equivalent fraction.

Can you show this pictorially?

Example 4 : Find the equivalent fraction of $\frac{15}{35}$ with denominator 7.

Solution : We have $\frac{15}{35} = \frac{\square}{7}$

We observe the denominator and find $35 \div 5 = 7$. We, therefore, divide both the numerator and the denominator of $\frac{15}{35}$ by 5.

$$\text{Thus, } \frac{15}{35} = \frac{15 \div 5}{35 \div 5} = \frac{3}{7}.$$

An interesting fact

Let us now note an interesting fact about equivalent fractions. For this, complete the given table. The first two rows have already been completed for you.

Equivalent fractions	Product of the numerator of the 1st and the denominator of the 2nd	Product of the numerator of the 2nd and the denominator of the 1st	Are the products equal?
$\frac{1}{3} = \frac{3}{9}$	$1 \times 9 = 9$	$3 \times 3 = 9$	Yes
$\frac{4}{5} = \frac{28}{35}$	$4 \times 35 = 140$	$5 \times 28 = 140$	Yes
$\frac{1}{4} = \frac{4}{16}$			
$\frac{2}{3} = \frac{10}{15}$			
$\frac{3}{7} = \frac{24}{56}$			

What do we infer? The product of the numerator of the first and the denominator of the second is equal to the product of denominator of the first and the numerator of the second in all these cases. These two products are called cross products. Work out the cross products for other pairs of equivalent fractions. Do you find any pair of fractions for which cross products are not equal? This rule is helpful in finding equivalent fractions.

Example 5 : Find the equivalent fraction of $\frac{2}{9}$ with denominator 63.

Solution : We have $\frac{2}{9} = \frac{\square}{63}$

For this, we should have, $9 \times \square = 2 \times 63$.

But $63 = 7 \times 9$, so $9 \times \square = 2 \times 7 \times 9 = 14 \times 9 = 9 \times 14$
or $9 \times \square = 9 \times 14$

By comparison, $\square = 14$. Therefore, $\frac{2}{9} = \frac{14}{63}$.

7.7 Simplest Form of a Fraction

Given the fraction $\frac{36}{54}$, let us try to get an equivalent fraction in which the numerator and the denominator have no common factor except 1.

How do we do it? We see that both 36 and 54 are divisible by 2.

$$\frac{36}{54} = \frac{36 \div 2}{54 \div 2} = \frac{18}{27}$$

But 18 and 27 also have common factors other than one.

The common factors are 1, 3, 9; the highest is 9.

$$\text{Therefore, } \frac{18}{27} = \frac{18 \div 9}{27 \div 9} = \frac{2}{3}$$



Now 2 and 3 have no common factor except 1; we say that the fraction $\frac{2}{3}$

is in the simplest form.

A fraction is said to be in the simplest (or lowest) form if its numerator and denominator have no common factor except 1.

The shortest way

The shortest way to find the equivalent fraction in the simplest form is to find the HCF of the numerator and denominator, and then divide both of them by the HCF.

A Game

The equivalent fractions given here are quite interesting. Each one of them uses all the digits from 1 to 9 once!

$$\frac{2}{6} = \frac{3}{9} = \frac{58}{174}$$

$$\frac{2}{4} = \frac{3}{6} = \frac{79}{158}$$

Try to find two more such equivalent fractions.

Consider $\frac{36}{24}$.

The HCF of 36 and 24 is 12.

Therefore, $\frac{36}{24} = \frac{36 \div 12}{24 \div 12} = \frac{3}{2}$. The

fraction $\frac{3}{2}$ is in the lowest form.

Thus, HCF helps us to reduce a fraction to its lowest form.

Try These

1. Write the simplest form of :

(i) $\frac{15}{75}$ (ii) $\frac{16}{72}$

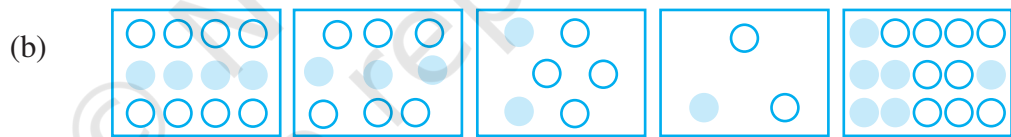
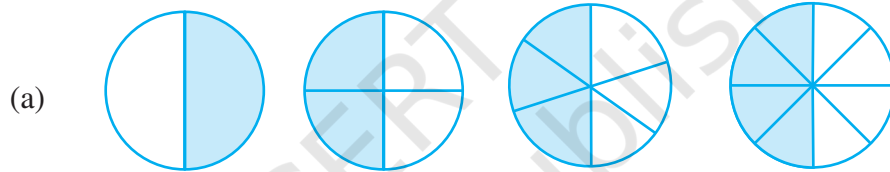
(iii) $\frac{17}{51}$ (iv) $\frac{42}{28}$ (v) $\frac{80}{24}$

2. Is $\frac{49}{64}$ in its simplest form?

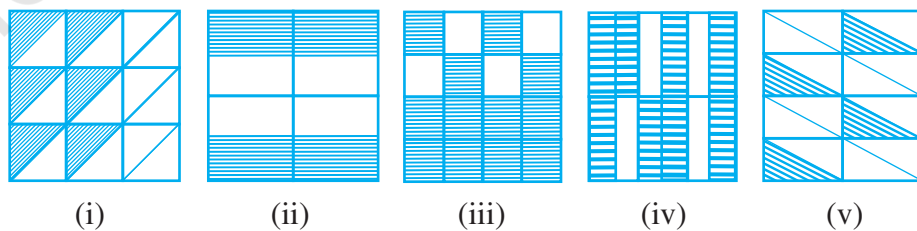
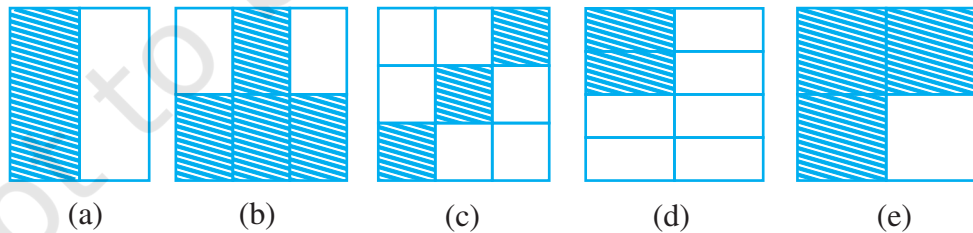


EXERCISE 7.3

1. Write the fractions. Are all these fractions equivalent?



2. Write the fractions and pair up the equivalent fractions from each row.



3. Replace \square in each of the following by the correct number :

(a) $\frac{2}{7} = \frac{8}{\square}$ (b) $\frac{5}{8} = \frac{10}{\square}$ (c) $\frac{3}{5} = \frac{\square}{20}$ (d) $\frac{45}{60} = \frac{15}{\square}$ (e) $\frac{18}{24} = \frac{\square}{4}$

4. Find the equivalent fraction of $\frac{3}{5}$ having

- (a) denominator 20 (b) numerator 9
(c) denominator 30 (d) numerator 27

5. Find the equivalent fraction of $\frac{36}{48}$ with

- (a) numerator 9 (b) denominator 4

6. Check whether the given fractions are equivalent :

(a) $\frac{5}{9}, \frac{30}{54}$ (b) $\frac{3}{10}, \frac{12}{50}$ (c) $\frac{7}{13}, \frac{5}{11}$

7. Reduce the following fractions to simplest form :

(a) $\frac{48}{60}$ (b) $\frac{150}{60}$ (c) $\frac{84}{98}$ (d) $\frac{12}{52}$ (e) $\frac{7}{28}$

8. Ramesh had 20 pencils, Sheelu had 50 pencils and Jamaal had 80 pencils. After 4 months, Ramesh used up 10 pencils, Sheelu used up 25 pencils and Jamaal used up 40 pencils. What fraction did each use up? Check if each has used up an equal fraction of her/his pencils?

9. Match the equivalent fractions and write two more for each.

(i) $\frac{250}{400}$ (a) $\frac{2}{3}$ (iv) $\frac{180}{360}$ (d) $\frac{5}{8}$
(ii) $\frac{180}{200}$ (b) $\frac{2}{5}$ (v) $\frac{220}{550}$ (e) $\frac{9}{10}$
(iii) $\frac{660}{990}$ (c) $\frac{1}{2}$

7.8 Like Fractions

Fractions with same denominators are called **like fractions**.

Thus, $\frac{1}{15}, \frac{2}{15}, \frac{3}{15}, \frac{8}{15}$ are all like fractions. Are $\frac{7}{27}$ and $\frac{7}{28}$ like fractions?

Their denominators are different. Therefore, they are not like fractions. They are called **unlike fractions**.

Write five pairs of like fractions and five pairs of unlike fractions.

7.9 Comparing Fractions

Sohni has $3\frac{1}{2}$ rotis in her plate and Rita has $2\frac{3}{4}$ rotis in her plate. Who has more rotis in her plate? Clearly, Sohni has 3 full rotis and more and Rita has less than 3 rotis. So, Sohni has more rotis.

Consider $\frac{1}{2}$ and $\frac{1}{3}$ as shown in Fig. 7.12. The portion of the whole corresponding to $\frac{1}{2}$ is clearly larger than the portion of the same whole corresponding to $\frac{1}{3}$.

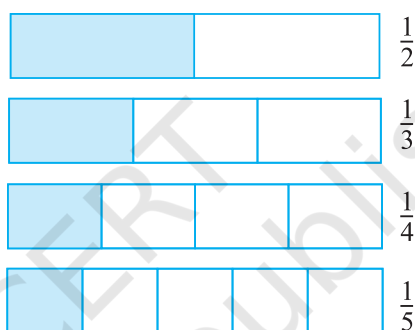


Fig 7.12

So $\frac{1}{2}$ is greater than $\frac{1}{3}$.

But often it is not easy to say which one out of a pair of fractions is larger. For example, which is greater, $\frac{1}{4}$ or $\frac{3}{10}$? For this, we may wish to show the fractions using figures (as in fig. 7.12), but drawing figures may not be easy especially with denominators like 13. We should therefore like to have a systematic procedure to compare fractions. It is particularly easy to compare like fractions. We do this first.

Try These

1. You get one-fifth of a bottle of juice and your sister gets one-third of the same size of a bottle of juice. Who gets more?

7.9.1 Comparing like fractions

Like fractions are fractions with the same denominator. Which of these are like fractions?

$$\frac{2}{5}, \frac{3}{4}, \frac{1}{5}, \frac{7}{2}, \frac{3}{5}, \frac{4}{5}, \frac{4}{7}$$



Let us compare two like fractions: $\frac{3}{8}$ and $\frac{5}{8}$.



In both the fractions the whole is divided into 8 equal parts. For $\frac{3}{8}$ and $\frac{5}{8}$, we take 3 and 5 parts respectively out of the 8 equal parts. Clearly, out of 8 equal parts, the portion corresponding to 5 parts is larger than the portion corresponding to 3 parts. Hence, $\frac{5}{8} > \frac{3}{8}$. Note the number of the parts taken is given by the numerator. It is, therefore, clear that for two fractions with the same denominator, the fraction with the greater numerator is greater. Between $\frac{4}{5}$ and $\frac{3}{5}$, $\frac{4}{5}$ is greater. Between $\frac{11}{20}$ and $\frac{13}{20}$, $\frac{13}{20}$ is greater and so on.

Try These

- Which is the larger fraction?
 (i) $\frac{7}{10}$ or $\frac{8}{10}$ (ii) $\frac{11}{24}$ or $\frac{13}{24}$ (iii) $\frac{17}{102}$ or $\frac{12}{102}$

Why are these comparisons easy to make?

- Write these in ascending and also in descending order.

- (a) $\frac{1}{8}, \frac{5}{8}, \frac{3}{8}$ (b) $\frac{1}{5}, \frac{11}{5}, \frac{4}{5}, \frac{3}{5}, \frac{7}{5}$ (c) $\frac{1}{7}, \frac{3}{7}, \frac{13}{7}, \frac{11}{7}, \frac{7}{7}$

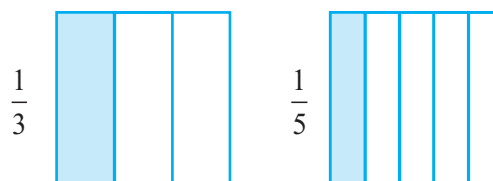
7.9.2 Comparing unlike fractions

Two fractions are unlike if they have different denominators. For example, $\frac{1}{3}$ and $\frac{1}{5}$ are unlike fractions. So are $\frac{2}{3}$ and $\frac{3}{5}$.

Unlike fractions with the same numerator :

Consider a pair of unlike fractions $\frac{1}{3}$ and $\frac{1}{5}$, in which the numerator is the same.

Which is greater $\frac{1}{3}$ or $\frac{1}{5}$?



In $\frac{1}{3}$, we divide the whole into 3 equal parts and take one. In $\frac{1}{5}$, we divide the whole into 5 equal parts and take one. Note that in $\frac{1}{3}$, the whole is divided into a smaller number of parts than in $\frac{1}{5}$. The equal part that we get in $\frac{1}{3}$ is, therefore, larger than the equal part we get in $\frac{1}{5}$. Since in both cases we take the same number of parts (i.e. one), the portion of the whole showing $\frac{1}{3}$ is larger than the portion showing $\frac{1}{5}$, and therefore $\frac{1}{3} > \frac{1}{5}$.

In the same way we can say $\frac{2}{3} > \frac{2}{5}$. In this case, the situation is the same as in the case above, except that the common numerator is 2, not 1. The whole is divided into a large number of equal parts for $\frac{2}{5}$ than for $\frac{2}{3}$. Therefore, each equal part of the whole in case of $\frac{2}{3}$ is larger than that in case of $\frac{2}{5}$. Therefore, the portion of the whole showing $\frac{2}{3}$ is larger than the portion showing $\frac{2}{5}$ and hence, $\frac{2}{3} > \frac{2}{5}$.

We can see from the above example that **if the numerator is the same in two fractions, the fraction with the smaller denominator is greater of the two.**

Thus, $\frac{1}{8} > \frac{1}{10}$, $\frac{3}{5} > \frac{3}{7}$, $\frac{4}{9} > \frac{4}{11}$ and so on.

Let us arrange $\frac{2}{1}, \frac{2}{13}, \frac{2}{9}, \frac{2}{5}, \frac{2}{7}$ in increasing order. All these fractions are unlike, but their numerator is the same. Hence, in such case, the larger the denominator, the smaller is the fraction. The smallest is $\frac{2}{13}$, as it has the largest denominator. The next three fractions in order are $\frac{2}{9}, \frac{2}{7}, \frac{2}{5}$. The greatest fraction is $\frac{2}{1}$ (It is with the smallest denominator). The arrangement in increasing order, therefore, is $\frac{2}{13}, \frac{2}{9}, \frac{2}{7}, \frac{2}{5}, \frac{2}{1}$.

Try These

1. Arrange the following in ascending and descending order :

(a) $\frac{1}{12}, \frac{1}{23}, \frac{1}{5}, \frac{1}{7}, \frac{1}{50}, \frac{1}{9}, \frac{1}{17}$

(b) $\frac{3}{7}, \frac{3}{11}, \frac{3}{5}, \frac{3}{2}, \frac{3}{13}, \frac{3}{4}, \frac{3}{17}$

(c) Write 3 more similar examples and arrange them in ascending and descending order.

Suppose we want to compare $\frac{2}{3}$ and $\frac{3}{4}$. Their numerators are different and so are their denominators. We know how to compare like fractions, i.e. fractions with the same denominator. We should, therefore, try to change the denominators of the given fractions, so that they become equal. For this purpose, we can use the method of equivalent fractions which we already know. Using this method we can change the denominator of a fraction without changing its value.

Let us find equivalent fractions of both $\frac{2}{3}$ and $\frac{3}{4}$.

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \dots \quad \text{Similarly, } \frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \dots$$

The equivalent fractions of $\frac{2}{3}$ and $\frac{3}{4}$ with the same denominator 12 are $\frac{8}{12}$ and $\frac{9}{12}$ respectively.

i.e. $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$. Since, $\frac{9}{12} > \frac{8}{12}$ we have, $\frac{3}{4} > \frac{2}{3}$.

Example 6 : Compare $\frac{4}{5}$ and $\frac{5}{6}$.

Solution : The fractions are unlike fractions. Their numerators are different too. Let us write their equivalent fractions.

$$\frac{4}{5} = \frac{8}{10} = \frac{12}{15} = \frac{16}{20} = \frac{20}{25} = \frac{24}{30} = \frac{28}{35} = \dots$$

and $\frac{5}{6} = \frac{10}{12} = \frac{15}{18} = \frac{20}{24} = \frac{25}{30} = \frac{30}{36} = \dots$

The equivalent fractions with the same denominator are :

$$\frac{4}{5} = \frac{24}{30} \text{ and } \frac{5}{6} = \frac{25}{30}$$

$$\text{Since, } \frac{25}{30} > \frac{24}{30} \text{ so, } \frac{5}{6} > \frac{4}{5}$$

Note that the common denominator of the equivalent fractions is 30 which is 5×6 . It is a common multiple of both 5 and 6.

So, when we compare two unlike fractions, we first get their equivalent fractions with a denominator which is a common multiple of the denominators of both the fractions.

Example 7 : Compare $\frac{5}{6}$ and $\frac{13}{15}$.

Solution : The fractions are unlike. We should first get their equivalent fractions with a denominator which is a common multiple of 6 and 15.

$$\text{Now, } \frac{5 \times 5}{6 \times 5} = \frac{25}{30}, \frac{13 \times 2}{15 \times 2} = \frac{26}{30}$$

$$\text{Since } \frac{26}{30} > \frac{25}{30} \text{ we have } \frac{13}{15} > \frac{5}{6}.$$

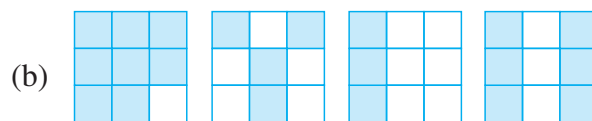
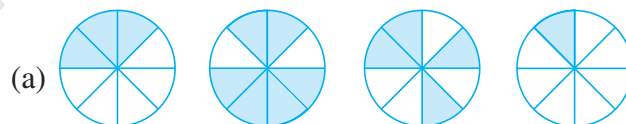
Why LCM?

The product of 6 and 15 is 90; obviously 90 is also a common multiple of 6 and 15. We may use 90 instead of 30; it will not be wrong. But we know that it is easier and more convenient to work with smaller numbers. So the common multiple that we take is as small as possible. This is why the LCM of the denominators of the fractions is preferred as the common denominator.



EXERCISE 7.4

- Write shaded portion as fraction. Arrange them in ascending and descending order using correct sign '<', '=', '>' between the fractions:



- (c) Show $\frac{2}{6}$, $\frac{4}{6}$, $\frac{8}{6}$ and $\frac{6}{6}$ on the number line. Put appropriate signs between the fractions given.

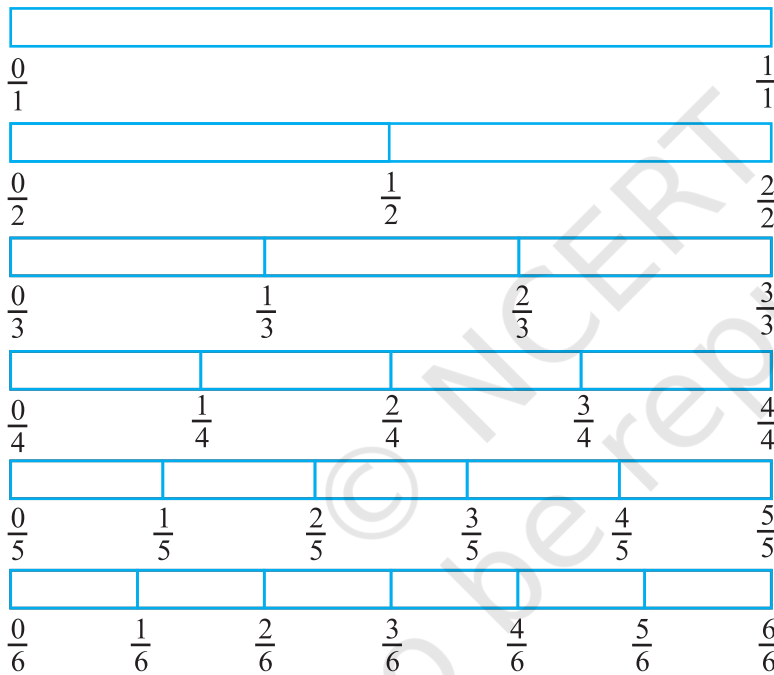
$$\frac{5}{6} \square \frac{2}{6}, \quad \frac{3}{6} \square 0, \quad \frac{1}{6} \square \frac{6}{6}, \quad \frac{8}{6} \square \frac{5}{6}$$

2. Compare the fractions and put an appropriate sign.

(a) $\frac{3}{6} \square \frac{5}{6}$ (b) $\frac{1}{7} \square \frac{1}{4}$ (c) $\frac{4}{5} \square \frac{5}{5}$ (d) $\frac{3}{5} \square \frac{3}{7}$

3. Make five more such pairs and put appropriate signs.

4. Look at the figures and write '<' or '>', '=' between the given pairs of fractions.



(a) $\frac{1}{6} \square \frac{1}{3}$ (b) $\frac{3}{4} \square \frac{2}{6}$ (c) $\frac{2}{3} \square \frac{2}{4}$ (d) $\frac{6}{6} \square \frac{3}{3}$ (e) $\frac{5}{6} \square \frac{5}{5}$

Make five more such problems and solve them with your friends.

5. How quickly can you do this? Fill appropriate sign. ('<', '=', '>')

(a) $\frac{1}{2} \square \frac{1}{5}$ (b) $\frac{2}{4} \square \frac{3}{6}$ (c) $\frac{3}{5} \square \frac{2}{3}$

(d) $\frac{3}{4} \square \frac{2}{8}$ (e) $\frac{3}{5} \square \frac{6}{5}$ (f) $\frac{7}{9} \square \frac{3}{9}$

(g) $\frac{1}{4} \square \frac{2}{8}$ (h) $\frac{6}{10} \square \frac{4}{5}$ (i) $\frac{3}{4} \square \frac{7}{8}$

(j) $\frac{6}{10} \square \frac{3}{5}$ (k) $\frac{5}{7} \square \frac{15}{21}$

6. The following fractions represent just three different numbers. Separate them into three groups of equivalent fractions, by changing each one to its simplest form.

(a) $\frac{2}{12}$ (b) $\frac{3}{15}$ (c) $\frac{8}{50}$ (d) $\frac{16}{100}$ (e) $\frac{10}{60}$ (f) $\frac{15}{75}$

(g) $\frac{12}{60}$ (h) $\frac{16}{96}$ (i) $\frac{12}{75}$ (j) $\frac{12}{72}$ (k) $\frac{3}{18}$ (l) $\frac{4}{25}$

7. Find answers to the following. Write and indicate how you solved them.

(a) Is $\frac{5}{9}$ equal to $\frac{4}{5}$? (b) Is $\frac{9}{16}$ equal to $\frac{5}{9}$?

(c) Is $\frac{4}{5}$ equal to $\frac{16}{20}$? (d) Is $\frac{1}{15}$ equal to $\frac{4}{30}$?

8. Ila read 25 pages of a book containing 100 pages. Lalita read $\frac{2}{5}$ of the same book. Who read less?

9. Rafiq exercised for $\frac{3}{6}$ of an hour, while Rohit exercised for $\frac{3}{4}$ of an hour. Who exercised for a longer time?

10. In a class A of 25 students, 20 passed with 60% or more marks; in another class B of 30 students, 24 passed with 60% or more marks. In which class was a greater fraction of students getting with 60% or more marks?

7.10 Addition and Subtraction of Fractions

So far in our study we have learnt about natural numbers, whole numbers and then integers. In the present chapter, we are learning about fractions, a different type of numbers.

Whenever we come across new type of numbers, we want to know how to operate with them. Can we combine and add them? If so, how? Can we take away some number from another? i.e., can we subtract one from the other? and so on. Which of the properties learnt earlier about the numbers hold now? Which are the new properties? We also see how these help us deal with our daily life situations.

Try These

1. My mother divided an apple into 4 equal parts. She gave me two parts and my brother one part. How much apple did she give to both of us together?
2. Mother asked Neelu and her brother to pick stones from the wheat. Neelu picked one fourth of the total stones in it and her brother also picked up one fourth of the stones. What fraction of the stones did both pick up together?
3. Sohan was putting covers on his note books. He put one fourth of the covers on Monday. He put another one fourth on Tuesday and the remaining on Wednesday. What fraction of the covers did he put on Wednesday?

Look at the following examples: A tea stall owner consumes in her shop $2\frac{1}{2}$ litres of milk in the morning and $1\frac{1}{2}$ litres of milk in the evening in preparing tea. What is the total amount of milk she uses in the stall?

Or Shekhar ate 2 chapatis for lunch and $1\frac{1}{2}$ chapatis for dinner. What is the total number of chapatis he ate?

Clearly, both the situations

require the fractions to be added. Some of these additions can be done orally and the sum can be found quite easily.

Do This

Make five such problems with your friends and solve them.

7.10.1 Adding or subtracting like fractions

All fractions cannot be added orally. We need to know how they can be added in different situations and learn the procedure for it. We begin by looking at addition of like fractions.

Take a 7×4 grid sheet (Fig 7.13). The sheet has seven boxes in each row and four boxes in each column.

How many boxes are there in total?

Colour five of its boxes in green.

What fraction of the whole is the green region?

Now colour another four of its boxes in yellow.

What fraction of the whole is this yellow region?

What fraction of the whole is coloured altogether?

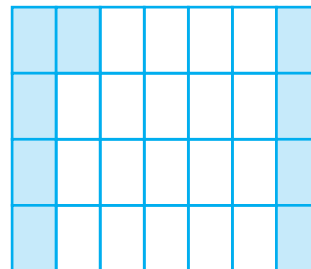


Fig 7.13

Does this explain that $\frac{5}{28} + \frac{4}{28} = \frac{9}{28}$?

Look at more examples

In Fig 7.14 (i) we have 2 quarter parts of the figure shaded. This means we have 2 parts out of 4 shaded or $\frac{1}{2}$ of the figure shaded.

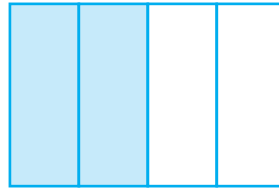


Fig. 7.14 (i)

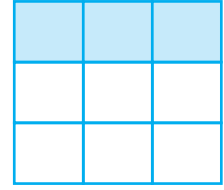


Fig. 7.14 (ii)

That is, $\frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$.

Look at Fig 7.14 (ii)

Fig 7.14 (ii) demonstrates $\frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{1+1+1}{9} = \frac{3}{9} = \frac{1}{3}$.

What do we learn from the above examples? The sum of two or more like fractions can be obtained as follows :

Step 1 Add the numerators.

Step 2 Retain the (common) denominator.

Step 3 Write the fraction as :

Result of Step 1

Result of Step 2

Let us, thus, add $\frac{3}{5}$ and $\frac{1}{5}$.

We have $\frac{3}{5} + \frac{1}{5} = \frac{3+1}{5} = \frac{4}{5}$

So, what will be the sum of $\frac{7}{12}$ and $\frac{3}{12}$?

Finding the balance

Sharmila had $\frac{5}{6}$ of a cake. She gave $\frac{2}{6}$ out of that to her younger brother. How much cake is left with her?

A diagram can explain the situation (Fig 7.15). (Note that, here the given fractions are like fractions).

We find that $\frac{5}{6} - \frac{2}{6} = \frac{5-2}{6} = \frac{3}{6}$ or $\frac{1}{2}$

(Is this not similar to the method of adding like fractions?)

Try These

1. Add with the help of a diagram.

(i) $\frac{1}{8} + \frac{1}{8}$ (ii) $\frac{2}{5} + \frac{3}{5}$ (iii) $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

2. Add $\frac{1}{12} + \frac{1}{12}$. How will we show this pictorially? Using paper folding?

3. Make 5 more examples of problems given in 1 and 2 above.

Solve them with your friends.





Fig 7.15

Thus, we can say that the difference of two like fractions can be obtained as follows:

Step 1 Subtract the smaller numerator from the bigger numerator.

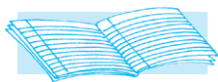
Step 2 Retain the (common) denominator.

Step 3 Write the fraction as : $\frac{\text{Result of Step 1}}{\text{Result of Step 2}}$

Can we now subtract $\frac{3}{10}$ from $\frac{8}{10}$?

Try These

1. Find the difference between $\frac{7}{8}$ and $\frac{3}{8}$.
2. Mother made a gud patti in a round shape. She divided it into 5 parts. Seema ate one piece from it. If I eat another piece then how much would be left?
3. My elder sister divided the watermelon into 16 parts. I ate 7 out them. My friend ate 4. How much did we eat between us? How much more of the watermelon did I eat than my friend? What portion of the watermelon remained?
4. Make five problems of this type and solve them with your friends.



EXERCISE 7.5

1. Write these fractions appropriately as additions or subtractions :

(a) ... =

(b) ... =

(c) ... =

2. Solve:

(a) $\frac{1}{18} + \frac{1}{18}$ (b) $\frac{8}{15} + \frac{3}{15}$ (c) $\frac{7}{7} - \frac{5}{7}$ (d) $\frac{1}{22} + \frac{21}{22}$ (e) $\frac{12}{15} - \frac{7}{15}$
 (f) $\frac{5}{8} + \frac{3}{8}$ (g) $1 - \frac{2}{3}$ ($1 = \frac{3}{3}$) (h) $\frac{1}{4} + \frac{0}{4}$ (i) $3 - \frac{12}{5}$

3. Shubham painted $\frac{2}{3}$ of the wall space in his room. His sister Madhavi helped and painted $\frac{1}{3}$ of the wall space. How much did they paint together?

4. Fill in the missing fractions.

(a) $\frac{7}{10} - \square = \frac{3}{10}$ (b) $\square - \frac{3}{21} = \frac{5}{21}$ (c) $\square - \frac{3}{6} = \frac{3}{6}$ (d) $\square + \frac{5}{27} = \frac{12}{27}$

5. Javed was given $\frac{5}{7}$ of a basket of oranges. What fraction of oranges was left in the basket?

7.10.2 Adding and subtracting fractions

We have learnt to add and subtract like fractions. It is also not very difficult to add fractions that do not have the same denominator. When we have to add or subtract fractions we first find equivalent fractions with the same denominator and then proceed.

What added to $\frac{1}{5}$ gives $\frac{1}{2}$? This means subtract $\frac{1}{5}$ from $\frac{1}{2}$ to get the required number.

Since $\frac{1}{5}$ and $\frac{1}{2}$ are unlike fractions, in order to subtract them, we first find their equivalent fractions with the same denominator. These are $\frac{2}{10}$ and $\frac{5}{10}$ respectively.

This is because $\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$ and $\frac{1}{5} = \frac{1 \times 2}{5 \times 2} = \frac{2}{10}$

Therefore, $\frac{1}{2} - \frac{1}{5} = \frac{5}{10} - \frac{2}{10} = \frac{5-2}{10} = \frac{3}{10}$

Note that 10 is the least common multiple (LCM) of 2 and 5.

Example 8 : Subtract $\frac{3}{4}$ from $\frac{5}{6}$.

Solution : We need to find equivalent fractions of $\frac{3}{4}$ and $\frac{5}{6}$, which have the



same denominator. This denominator is given by the LCM of 4 and 6. The required LCM is 12.

$$\text{Therefore, } \frac{5}{6} - \frac{3}{4} = \frac{5 \times 2}{6 \times 2} - \frac{3 \times 3}{4 \times 3} = \frac{10}{12} - \frac{9}{12} = \frac{1}{12}$$

Example 9 : Add $\frac{2}{5}$ to $\frac{1}{3}$.

Solution : The LCM of 5 and 3 is 15.

$$\text{Therefore, } \frac{2}{5} + \frac{1}{3} = \frac{2 \times 3}{5 \times 3} + \frac{1 \times 5}{3 \times 5} = \frac{6}{15} + \frac{5}{15} = \frac{11}{15}$$

Example 10 : Simplify $\frac{3}{5} - \frac{7}{20}$

Solution : The LCM of 5 and 20 is 20.

$$\begin{aligned} \text{Therefore, } \frac{3}{5} - \frac{7}{20} &= \frac{3 \times 4}{5 \times 4} - \frac{7}{20} = \frac{12}{20} - \frac{7}{20} \\ &= \frac{12-7}{20} = \frac{5}{20} = \frac{1}{4} \end{aligned}$$

Try These 

1. Add $\frac{2}{5}$ and $\frac{3}{7}$.

2. Subtract $\frac{2}{5}$ from $\frac{5}{7}$.

How do we add or subtract mixed fractions?

Mixed fractions can be written either as a whole part plus a proper fraction or entirely as an improper fraction. One way to add (or subtract) mixed fractions is to do the operation separately for the whole parts and the other way is to write the mixed fractions as improper fractions and then directly add (or subtract) them.

Example 11 : Add $2\frac{4}{5}$ and $3\frac{5}{6}$

Solution : $2\frac{4}{5} + 3\frac{5}{6} = 2 + \frac{4}{5} + 3 + \frac{5}{6} = 5 + \frac{4}{5} + \frac{5}{6}$

$$\text{Now } \frac{4}{5} + \frac{5}{6} = \frac{4 \times 6}{5 \times 6} + \frac{5 \times 5}{6 \times 5} \quad (\text{Since LCM of 5 and 6} = 30)$$

$$= \frac{24}{30} + \frac{25}{30} = \frac{49}{30} = \frac{30+19}{30} = 1 + \frac{19}{30}$$

$$\text{Thus, } 5 + \frac{4}{5} + \frac{5}{6} = 5 + 1 + \frac{19}{30} = 6 + \frac{19}{30} = 6\frac{19}{30}$$

$$\text{And, therefore, } 2\frac{4}{5} + 3\frac{5}{6} = 6\frac{19}{30}$$

Think, discuss and write

Can you find the other way of doing this sum?

Example 12 : Find $4\frac{2}{5} - 2\frac{1}{5}$

Solution : The whole numbers 4 and 2 and the fractional numbers $\frac{2}{5}$ and $\frac{1}{5}$

can be subtracted separately. (Note that $4 > 2$ and $\frac{2}{5} > \frac{1}{5}$)

$$\text{So, } 4\frac{2}{5} - 2\frac{1}{5} = (4 - 2) + \left(\frac{2}{5} - \frac{1}{5}\right) = 2 + \frac{1}{5} = 2\frac{1}{5}$$

Example 13 : Simplify: $8\frac{1}{4} - 2\frac{5}{6}$

Solution : Here $8 > 2$ but $\frac{1}{4} < \frac{5}{6}$. We proceed as follows:

$$8\frac{1}{4} = \frac{(8 \times 4) + 1}{4} = \frac{33}{4} \quad \text{and} \quad 2\frac{5}{6} = \frac{2 \times 6 + 5}{6} = \frac{17}{6}$$

$$\begin{aligned} \text{Now, } \frac{33}{4} - \frac{17}{6} &= \frac{33 \times 3}{12} - \frac{17 \times 2}{12} \quad (\text{Since LCM of 4 and 6} = 12) \\ &= \frac{99 - 34}{12} = \frac{65}{12} = 5\frac{5}{12} \end{aligned}$$



EXERCISE 7.6

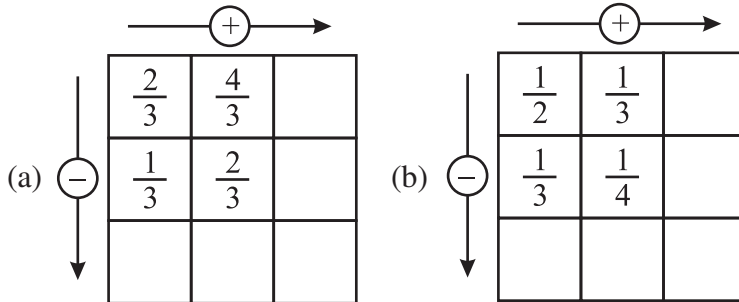
1. Solve

- (a) $\frac{2}{3} + \frac{1}{7}$ (b) $\frac{3}{10} + \frac{7}{15}$ (c) $\frac{4}{9} + \frac{2}{7}$ (d) $\frac{5}{7} + \frac{1}{3}$ (e) $\frac{2}{5} + \frac{1}{6}$
- (f) $\frac{4}{5} + \frac{2}{3}$ (g) $\frac{3}{4} - \frac{1}{3}$ (h) $\frac{5}{6} - \frac{1}{3}$ (i) $\frac{2}{3} + \frac{3}{4} + \frac{1}{2}$ (j) $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$
- (k) $1\frac{1}{3} + 3\frac{2}{3}$ (l) $4\frac{2}{3} + 3\frac{1}{4}$ (m) $\frac{16}{5} - \frac{7}{5}$ (n) $\frac{4}{3} - \frac{1}{2}$

2. Sarita bought $\frac{2}{5}$ metre of ribbon and Lalita $\frac{3}{4}$ metre of ribbon. What is the total length of the ribbon they bought?

3. Naina was given $1\frac{1}{2}$ piece of cake and Najma was given $1\frac{1}{3}$ piece of cake. Find the total amount of cake was given to both of them.

4. Fill in the boxes : (a) $\square - \frac{5}{8} = \frac{1}{4}$ (b) $\square - \frac{1}{5} = \frac{1}{2}$ (c) $\frac{1}{2} - \square = \frac{1}{6}$
5. Complete the addition-subtraction box.



6. A piece of wire $\frac{7}{8}$ metre long broke into two pieces. One piece was $\frac{1}{4}$ metre long. How long is the other piece?
7. Nandini's house is $\frac{9}{10}$ km from her school. She walked some distance and then took a bus for $\frac{1}{2}$ km to reach the school. How far did she walk?
8. Asha and Samuel have bookshelves of the same size partly filled with books. Asha's shelf is $\frac{5}{6}$ th full and Samuel's shelf is $\frac{2}{5}$ th full. Whose bookshelf is more full? By what fraction?
9. Jaidev takes $2\frac{1}{5}$ minutes to walk across the school ground. Rahul takes $\frac{7}{4}$ minutes to do the same. Who takes less time and by what fraction?

What have we discussed?

1. (a) A fraction is a number representing a part of a whole. The whole may be a single object or a group of objects.
 (b) When expressing a situation of counting parts to write a fraction, it must be ensured that all parts are equal.
2. In $\frac{5}{7}$, 5 is called the numerator and 7 is called the denominator.
3. Fractions can be shown on a number line. Every fraction has a point associated with it on the number line.
4. In a proper fraction, the numerator is less than the denominator. The fractions, where the numerator is greater than the denominator are called improper fractions. An improper fraction can be written as a combination of a whole and a part, and such fraction then called mixed fractions.
5. Each proper or improper fraction has many equivalent fractions. To find an equivalent fraction of a given fraction, we may multiply or divide both the numerator and the denominator of the given fraction by the same number.
6. A fraction is said to be in the simplest (or lowest) form if its numerator and the denominator have no common factor except 1.



Note

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